Conductivity of electronic liquid-crystalline mesophases

Rafael M. Fernandes,[1,2,3,*] Jörg Schmalian,[1] and Harry Westfahl, Jr.[2]

[1]*Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*
[2]*Laboratório Nacional de Luz Síncrotron, Caixa Postal 6192, 13083-970 Campinas, SP, Brazil*
[3]*Instituto de Física "Gleb Wataghin," Universidade Estadual de Campinas, 13083-970 Campinas, SP, Brazil*

We investigate the connection between the transport properties and the thermodynamics of electronic systems with a tendency to form broken-symmetry mesophases evocative of the physics of liquid crystals. Through a hydrodynamic approach to the electronic transport in inhomogeneous systems, we develop a perturbative expansion for the macroscopic conductivity to study the transport of two-dimensional smectic and nematic phases. At the fluctuation-induced first-order phase transition expected for the smectic to isotropic transition, a jump in the macroscopic conductivity is predicted, with a directional dependence that reflects the fluctuation spectrum of the order parameter. When elastic fluctuation modes melt the smectic phase into a nematic phase, the resultant nematic order parameter is shown to be linearly proportional to the conductivity anisotropy. We also outline qualitative comparisons with recent experimental works on strongly correlated materials that show evidences of electronic liquid-crystalline mesophases.

PACS number(s): 72.10.Bg, 71.27.+a, 05.65.+b, 64.70.M−

I. INTRODUCTION

Recent investigations on strongly correlated electron systems suggest the emergence of inhomogeneous charge-ordered phases reminiscent from the smectic and nematic states commonly found in liquid crystals.[1] Such an analogy makes reference to the broken symmetries of each phase. While the nematic phase breaks only rotational symmetry, the smectic state also breaks the translational invariance along one particular direction. Such electronic mesophases can be envisaged as "fluctuating" and static charge stripes, respectively. Numerous experimental works provide evidences for the occurrence of smectic charge ordering in nickelates,[2,3] manganites,[4] and $La_{2-x}Ba_xCuO_4$ (LBCO) cuprates,[5] as well as indications of the existence of nematic order in quantum Hall systems,[6] ruthenates,[7] $La_{2-x}Sr_xCuO_4$ (LSCO),[8,9] and $YBa_2Cu_3O_{6+y}$ (YBCO) cuprates.[10,11]

The order of these mesophases is directly manifested in anisotropies of the electric transport as well as in the magnetic or charge response of the system, similar to the anisotropic optical and hydrodynamic properties of liquid crystals, such as birefringence and viscosity.[12] An interesting question is the explicit connection between the order parameters of these inhomogeneous mesophases and their transport properties. While it is natural to expect that the resistivity anisotropy is directly linked to an order parameter that breaks rotational or translational symmetry, it is less obvious how to connect transport properties to spatial or temporal fluctuations and correlations of the system.

In this paper, we investigate the transport properties of inhomogeneous electronic mesophases in the hydrodynamic transport limit, where collisions dominate. In this regime, electron transport can be understood as a diffusive process with inhomogeneous and potentially time-dependent diffusion constant $D(\mathbf{x},t)$. We use a coarse-grained description and assume that the dephasing length scale l_0, beyond which diffusive transport sets in, is small compared to the length scale ξ on which the inhomogeneities of the electronic me-

sophases vary. The condition $l_0 \ll \xi$ is expected to be valid close to finite temperature phase transitions. This is certainly true close to second-order transitions where ξ diverges. However, even if the transition is weakly first-order, we still expect ξ to be larger than the lattice constant a while l_0 becomes comparable to a due to strong inelastic scattering close to the transition.

As will be discussed in detail below, our formalism for the conductivity of inhomogeneous mesophases has a close connection to the theory of random resistor network (RRN).[13] Previously, the classical RRN was applied to explain transport properties of composite films,[14] manganites,[15] and silver chalcogenides.[16] Moreover, numerical simulations of correlated versions of this network were carried out in the contexts of manganites,[17] disordered electronic nematic phases in cuprates,[18] and finite temperature Mott transitions.[19] The present approach is an analytical theory for transport in electronic mesophases valid for inhomogeneities that are small in amplitude but well correlated in space. It can also be considered as an analytical theory for resistor networks with correlated local conductivities.[20] Here, the spatial correlations between the microscopic resistors are directly connected to the correlations of the order parameter describing the mesophase and not to an arbitrary distribution function.[21] In the formalism that we develop, not only the order-parameter mean value but also its relevant fluctuations are explicit related to the macroscopic dc conductivity through a perturbative expansion.

By applying this general hydrodynamic transport model to the static charge-striped phase, we show that while the conductivity measured perpendicular to the stripes probes mainly the order-parameter amplitude, the conductivity measured parallel to them is particularly sensitive to the fluctuation spectrum. Thus, from transport measurements, it is in principle possible to obtain information about the microscopic character of the anisotropic mesophase, such as the first moments of the Boltzmann distribution function, for example. As a specific realization of the thermodynamics of the electronic smectic state, we consider the Coulomb-frustrated

Ising model first introduced by Emery and Kivelson[22,23] in the context of doped Mott insulators that present high-temperature superconductivity. We obtain an analytic expression for the dc conductivity by using the self-consistent mean-field solution of the effective Ginzburg-Landau Hamiltonian called the Brazovskii Hamiltonian.[24] We then show that the fluctuation-induced first-order transition from the isotropic liquid to the smectic phase is manifested as an anisotropic jump of the conductivity, whose sign brings information about the fluctuation spectrum of the order parameter. Qualitative comparisons to experimental data showing jumps in the resistivity of nickelates are also outlined.

In our theory, the investigation of the transport properties of the electronic nematics focuses on the role of thermally excited modes. Thus, it is complementary to the analysis of quantum modes as discussed in the recent literature.[25] Based on the work of Toner and Nelson,[26] we also describe the high-temperature nematic phase as a smectic phase melted due to the elastic fluctuations of stripes. We show explicitly that there is a temperature range where the macroscopic conductivity anisotropy is linearly proportional to the nematic order parameter, as expected from symmetry considerations,[27] and determine the prefactor that connects transport properties and nematic order. The same linear relation including the prefactor were also observed in numerical simulations of a *disordered* electronic nematic phase at zero temperature,[18] suggesting a close connection between the two rather different approaches. Finally, a very recent experiment regarding YBCO showed a remarkable resemblance between the spectral weight of the low-energy anisotropic spin fluctuations obtained through neutron scattering and the resistivity anisotropy obtained from transport measurements.[11]

The remainder of the paper is organized as follows. In Sec. II, we provide a detailed derivation of the hydrodynamic model for the diffusion of an electron in an inhomogeneous medium, which will be used throughout the work. Section III is devoted to the application of this formalism to the transport properties of an electronic smectic phase. Not only do we outline very general properties but we also obtain specific results after describing the charge stripes thermodynamics by the Brazovskii model. In Sec. IV, the dc conductivity of an electronic phase with nematic type of order is investigated through the hydrodynamic transport model. Such a state is described as a smectic phase melted by thermally excited elastic fluctuation modes of the stripes. Comparisons to other approaches as well as to recent experiments involving doped transition-metal oxides are delineated. Finally, Sec. V is devoted to the final remarks and acknowledgments.

II. DIFFUSIVE TRANSPORT IN INHOMOGENEOUS MESOPHASES

Let us consider the diffusion of an electron in an arbitrary inhomogeneous medium. We start from the continuity equation,

$$\frac{\partial \rho(\mathbf{x},t)}{\partial t} + \nabla \cdot \mathbf{j}(\mathbf{x},t) = 0, \tag{1}$$

connecting the charge and current densities. The macroscopic conductivity of the medium $\sigma_{\alpha\beta}$ relates the mean current density to the external electric field $-\nabla U_{\text{ext}}(\mathbf{x},t)$,

$$\langle j_\alpha(\mathbf{x},t)\rangle = -\int \sigma_{\alpha\beta}(t-t')\nabla_\beta U_{\text{ext}}(\mathbf{x},t')dt', \tag{2}$$

yielding, in the Fourier space,

$$\langle \rho(\mathbf{k},\omega)\rangle = -i\frac{\sigma_{\alpha\beta}(\omega)k_\alpha k_\beta}{\omega}U_{\text{ext}}(\mathbf{k},\omega). \tag{3}$$

Next, we assume that locally the diffusive relation

$$j_\alpha(\mathbf{x},t) = -\chi_0 D_{\alpha\beta}(\mathbf{x},t)\nabla_\beta U_{\text{loc}}(\mathbf{x},t) \tag{4}$$

holds for the current density in terms of the local electrical field $-\nabla U_{\text{loc}}(\mathbf{x},t)$, where $D_{\alpha\beta}(\mathbf{x},t)$ denotes the inhomogeneous and possible time-dependent local diffusion coefficient. Within linear response, the uniform charge susceptibility χ_0 connects the local charge variations with the difference between the local and external potentials

$$\rho(\mathbf{x},t) = \chi_0[U_{\text{loc}}(\mathbf{x},t) - U_{\text{ext}}(\mathbf{x},t)]. \tag{5}$$

Combining the last two expressions, we are able to relate the external and local potentials via

$$\hat{G}^{-1}U_{\text{loc}}(\mathbf{x},t) = \frac{\partial U_{\text{ext}}(\mathbf{x},t)}{\partial t}, \tag{6}$$

with inverse diffusion operator

$$\hat{G}^{-1} = \frac{\partial}{\partial t} - \nabla_\alpha[D_{\alpha\beta}(\mathbf{x},t)\nabla_\beta]. \tag{7}$$

This yields for the charge density

$$\rho(\mathbf{x},t) = \chi_0\left[\hat{G}\frac{\partial}{\partial t} - 1\right]U_{\text{ext}}(\mathbf{x},t). \tag{8}$$

After taking the configurational average over $\rho(\mathbf{x},t)$, we can Fourier transform its mean value and obtain, by comparing to Eq. (3),

$$\sigma_{\alpha\beta}(\omega) = \lim_{k\to 0}\frac{i\omega}{k_\alpha k_\beta}\chi_0[-i\omega G(\mathbf{k},\omega) - 1], \tag{9}$$

where $G(\mathbf{k},\omega)$ is the Fourier transform of the average of the differential operator (7). In the dc limit, this finally yields

$$\sigma_{\alpha\beta} = \chi_0 \lim_{\omega\to 0}\lim_{k\to 0}\frac{\omega^2}{k_\alpha k_\beta}\text{Re}[G(\mathbf{k},\omega)]. \tag{10}$$

Using the Einstein relation,[28] we can identify the anisotropic macroscopic diffusion coefficient $D_{\alpha\beta}^{\text{macro}}$ as

$$D_{\alpha\beta}^{\text{macro}} = \lim_{\omega\to 0}\lim_{k\to 0}\frac{\omega^2}{k_\alpha k_\beta}\text{Re}[G(\mathbf{k},\omega)].$$

Therefore, it is clear that even if the tensor of the local diffusion coefficient behaves as $D_{\alpha\beta}(\mathbf{x},t) = D(\mathbf{x},t)\delta_{\alpha\beta}$, the global diffusion coefficient can be anisotropic as long as $D(\mathbf{x},t)$ is a function of the direction of $\mathbf{x}$.

This hydrodynamic formalism has a one-to-one analogy with the theory of RRN.[29] In its most elementary form, one considers two resistors with conductivities σ_A and σ_B randomly distributed with probabilities p and $1-p$ over the

links of a network. An external electric potential U_{ext} is then applied in each site through a local capacitor with specific capacitance C. As shown by Ref. 29, the macroscopic conductivity is given by a result identical to Eq. (10), with C playing the role of the charge susceptibility while the local conductivities of the RRN correspond to $\chi_0 D(\mathbf{x})$. In the RRN problem, the average is performed over the distribution function of the resistor network, which is frequently assumed to be a binomial distribution, characterized by the probability p.

The key difference between our approach and the RRN theory is that the distribution function for the local diffusion coefficient is determined by the distribution function of the order parameter. Let the order of an inhomogeneous electronic nematic or smectic state be characterized by a scalar density field $\rho(\mathbf{x},t)$. Here, $\rho(\mathbf{x},t)$ is the deviation of the coarse-grained electron density from its mean value. We then assume a simple connection

$$D_{\alpha\beta}(\mathbf{x},t) \equiv D_{\alpha\beta}[\rho(\mathbf{x},t)] \tag{11}$$

between the spatially varying diffusion coefficient and the electron density of the electronic mesophase, meaning that the temporal and spatial variations of the diffusion coefficient are determined solely by those of $\rho(\mathbf{x},t)$. In our cases of interest, the ordered inhomogeneous state is characterized by an order parameter that varies in space alternating between $\rho > 0$ and $\rho < 0$. For instance, in electronic smectics, $\rho > 0$ ($\rho < 0$) denotes a hole-rich (poor) coarse-grained region. Since each of these regions has its own conducting properties, we can associate different local conductivities to each of them.

For weakly inhomogeneous systems, we expand $D_{\alpha\beta}(\mathbf{x},t)$ relative to the homogeneous state, where $\rho(\mathbf{x},t)=0$. Hence, we propose the following specific form for $D_{\alpha\beta}(\mathbf{x},t)$ of Eq. (11):

$$D_{\alpha\beta}(\mathbf{x},t) = \chi_0 \sigma_0 [1 + g\rho(\mathbf{x},t)]\delta_{\alpha\beta}, \tag{12}$$

where σ_0 is the microscopic conductivity mean value and g is the coupling constant measuring the contrast between the conductivities of distinct regions. The physical meaning of these two parameters can be better visualized by considering the limit of an ordered *homogeneous* phase where $\rho = \pm\rho_0$. Denoting the conductivity of the saturated homogeneous $\rho > 0$ ($\rho < 0$) phase as $\sigma_>(\sigma_<)$, we have $\sigma_0 = (\sigma_> + \sigma_<)/2$ and $g = \rho_0^{-1}(\sigma_> - \sigma_<)/(\sigma_> + \sigma_<)$. A relation similar to Eq. (12) was considered in the context of the Mott transition by Papanikolaou *et al.*[19] However, in that case it was necessary to include a next-order term $\propto\rho(\mathbf{x},t)^2$ in the expansion (12) originated from the contribution of interface scattering. Such effects are not included in the present work.

Now, we particularize our analysis to the situation where the inhomogeneities of the system are small, such that $g \ll 1$. Thus, the diffusion differential operator (7) can be perturbatively expanded as $\hat{G} = \Sigma_{n=0}^{\infty} g^n (\hat{G}_0 \hat{V})^n \hat{G}_0$, with

$$(\hat{G}_0)^{-1} = \frac{\partial}{\partial t} - D_0\nabla^2$$

$$\hat{V} = D_0 \nabla \cdot [\rho(\mathbf{x},t)\nabla], \tag{13}$$

where the bare diffusion coefficient is $D_0 = \chi_0\sigma_0$. In order to obtain a perturbative expansion for the conductivity through Eq. (10), we need to perform an average over ρ before taking the Fourier transform of $\hat{V}$. There are two opposite limits one can consider. In the first one, which we shall call *quenched* limit, the electron diffuses faster than the field fluctuates, probing a frozen configuration of the order parameter. Hence, after taking the proper Fourier transforms, we obtain, to the second order in g,

$$\sigma_{\alpha\alpha} = \sigma_0 \Bigg[1 + g\langle\rho(\mathbf{k}=\mathbf{0},\omega=0)\rangle$$
$$- g^2 \int d^dk\, d\omega \frac{k^4(\hat{\mathbf{n}}_\alpha \cdot \hat{\mathbf{k}})^2}{(\omega^2 D_0^{-2} + k^4)} \langle\rho(\mathbf{k},\omega)\rho(-\mathbf{k},-\omega)\rangle \Bigg], \tag{14}$$

where $\hat{n}_\alpha$ is the direction taken to measure the conductivity and $\langle\cdots\rangle$ denotes the proper average over the order parameter, which can be the usual thermodynamic average, for example. Similarly we could also consider transport in nonequilibrium configurations such as glassy states; then, $\langle\cdots\rangle$ refers to the corresponding dynamic average of the nonequilibrium configuration under consideration.[30,31] For future reference, we rewrite the formula (14) for the special situation where the field is static

$$\sigma_{\alpha\alpha} = \sigma_0 \Bigg[1 + g\langle\rho(\mathbf{k}=\mathbf{0})\rangle - g^2 \int d^dk (\hat{\mathbf{n}}_\alpha \cdot \hat{\mathbf{k}})^2 \langle\rho(\mathbf{k})\rho(-\mathbf{k})\rangle \Bigg]. \tag{15}$$

In the opposite limit, which we shall call *annealed* limit, we consider a field that fluctuates much faster than the electron diffuses. Therefore, it is legitimate to replace the order parameter by its mean value $\rho \to \langle\rho\rangle$ in the microscopic conductivity expression (12), yielding

$$\sigma_{\alpha\alpha} = \sigma_0 \Bigg[1 + g\langle\rho(\mathbf{k}=\mathbf{0})\rangle - g^2 \int d^dk (\hat{\mathbf{n}}_\alpha \cdot \hat{\mathbf{k}})^2 \langle\rho(\mathbf{k})\rangle\langle\rho(-\mathbf{k})\rangle \Bigg]. \tag{16}$$

It is clear that the anisotropic character of the conductivity is not manifested until the second order in g. Moreover, this second-order term reflects a fundamental difference between the quenched and annealed limits. While in the latter the conductivity behavior is dictated by the order-parameter amplitude, in the former we note a tight connection between the macroscopic conductivity $\sigma_{\alpha\alpha}$ and the correlation function $\langle\rho(\mathbf{k})\rho(-\mathbf{k})\rangle$, resembling other similar relations found in condensed-matter systems, such as between the scattering cross section and the thermodynamic correlation functions.

III. TRANSPORT IN THE SMECTIC PHASE

Using the formalism developed in Sec. II, there are some very general statements we can make about the conductivity of an electronic smectic mesophase, independently of the

specific model under consideration. Hereafter, we consider that the static order parameter $\rho(\mathbf{x})$ describes local fluctuations of the charge density and that the electron diffuses much faster than the field changes (quenched limit). Since the system is usually electrically neutral, such that $\langle \rho(\mathbf{k} = \mathbf{0}) \rangle = \frac{1}{V} \int \langle \rho(\mathbf{x}) \rangle d^d x = 0$, the lowest nonvanishing correction to the uniform conductivity in perturbation theory is of second order in the contrast. After writing the correlation function as

$$\langle \rho(\mathbf{k}) \rho(-\mathbf{k}) \rangle = \mathcal{C}(\mathbf{k}) + \langle \rho(\mathbf{k}) \rangle \langle \rho(-\mathbf{k}) \rangle, \qquad (17)$$

where $\mathcal{C}(\mathbf{k})$ is the connected correlation function, we can split the second-order term of the macroscopic conductivity (15) in two parts $\sigma_{\alpha\alpha} = \sigma_0 [1 - g^2(\sigma'_{\alpha\alpha} + \sigma''_{\alpha\alpha})]$,

$$\sigma'_{\alpha\alpha} = \int d^d k (\hat{\mathbf{n}}_\alpha \cdot \hat{\mathbf{k}})^2 \mathcal{C}(\mathbf{k})$$

$$\sigma''_{\alpha\alpha} = \int d^d k (\hat{\mathbf{n}}_\alpha \cdot \hat{\mathbf{k}})^2 \langle \rho(\mathbf{k}) \rangle \langle \rho(-\mathbf{k}) \rangle. \qquad (18)$$

In the case of interest here, the correlation function depends only on the modulus of the momentum, i.e., $\mathcal{C}(\mathbf{k}) = \mathcal{C}(k)$. Therefore, $\sigma'_{\alpha\alpha}$ does not depend on the direction taken to measure the conductivity since $\hat{\mathbf{n}}_\alpha$ will be integrated out over all directions. Hence, $\sigma'_{\alpha\alpha}$ is isotropic and depends only on the order-parameter fluctuation spectrum

$$\sigma'_{\alpha\alpha} \propto \int d^d k \mathcal{C}(k) = \mathcal{C}(\mathbf{x}, \mathbf{x}) = \langle \rho^2(\mathbf{x}) \rangle - \langle \rho(\mathbf{x}) \rangle^2. \quad (19)$$

Meanwhile, the term $\sigma''_{\alpha\alpha}$ is proportional to the averaged order parameter. For small amplitude static stripes, it is reasonable to assume that the order-parameter mean value is strongly anisotropic and has a pronounced peak along the modulation direction $\mathbf{q_0}$, such that $\langle \rho(\mathbf{k}) \rangle \sim A \delta(\mathbf{k} - \mathbf{q_0})$. Higher harmonics [entering as $\delta(\mathbf{k} - 2\mathbf{q_0})$ or $\delta(\mathbf{k} - 3\mathbf{q_0})$, etc.] only matter once the amplitude of the inhomogeneity becomes large, i.e., $\rho(\mathbf{x}, t)$ becomes large compared to the mean electron density. It follows that unlike $\sigma'_{\alpha\alpha}$, the term $\sigma''_{\alpha\alpha}$ is anisotropic and depends on the relative angle θ_α between the external electric field and the modulation vector through

$$\sigma''_{\alpha\alpha} \propto A^2 \cos^2 \theta_\alpha. \qquad (20)$$

Thus, if the conductivity is measured perpendicular to the stripes ($\hat{\mathbf{n}}_\alpha \parallel \mathbf{q_0}$), the order parameter and its fluctuation spectrum will be probed. However, if the current is applied parallel to the stripes ($\hat{\mathbf{n}}_\alpha \perp \mathbf{q_0}$), the measurement will be sensitive solely to the fluctuations of the order parameter. Clearly, the transport quantity that only probes the order-parameter amplitude is the anisotropic conductivity $\sigma_\perp - \sigma_\parallel$.

Another interesting situation to investigate is when the system undergoes a phase transition to the state with no broken symmetries: the isotropic liquid. For instance, let us consider a first-order transition from the liquid to the smectic phase. If the conductivity is measured parallel to the stripes direction, it is expected that $\sigma''_{\alpha\alpha}$ vanishes and the isotropic part $\sigma'_{\alpha\alpha}$ will dictate the behavior of the dc conductivity. Since fluctuations of the order parameter are usually larger in the liquid-disordered state, we expect from Eq. (19) that

close to the transition, the conductivity of the liquid phase will be smaller than the one of the ordered smectic phase $\sigma_\parallel^{\mathrm{liq}} < \sigma_\parallel^{\mathrm{smc}}$.

Instead, if the conductivity is measured perpendicular to the stripes direction, the contribution of the term $\sigma''_{\alpha\alpha}$ will be maximum. Unlike its counterpart $\sigma'_{\alpha\alpha}$, this term is greater in the ordered side, since the order-parameter amplitude vanishes in the liquid phase. Hence, the question of whether $\sigma_\perp^{\mathrm{dis}}$ or $\sigma_\perp^{\mathrm{smc}}$ is larger close to the phase transition depends on the ratio between the jumps of the fluctuations and of the amplitude of the order parameter. For strong first-order transitions, we expect the latter to be more significant, meaning that $\sigma_\perp^{\mathrm{liq}} > \sigma_\perp^{\mathrm{smc}}$.

Let us now apply the hydrodynamic transport formalism to a specific model for the thermodynamics of the electronic smectics. In particular, we focus on doped-layered transition-metal oxides, which are believed to be well described as two-dimensional (2D)-doped Mott insulators.[32] In these compounds, the stripes appear as a compromise between a tendency of the doped charges to phase separate from the antiferromagnetic background spins and the long-range electrostatic repulsion between alike charges.[22] A simple model that attempts to capture these properties is the Coulomb-frustrated Ising model[23]

$$\mathcal{H}_{\text{Ising-Coulomb}} = \frac{1}{2} \int d^2 x \left\{ \tau_0 \rho^2 + |\nabla \rho|^2 + \frac{u}{4} \rho^4 \right\} \\ + \frac{q_0^3}{2\pi} \int d^2 x \frac{\rho(\mathbf{x}) \rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}, \qquad (21)$$

where $\rho(\mathbf{x}) > 0$ denotes a hole-rich region (larger conductivity) while $\rho(\mathbf{x}) < 0$ denotes a hole-poor region (smaller conductivity), with $\int \langle \rho(\mathbf{x}) \rangle d^2 x = 0$. As usual, τ_0 denotes the reduced temperature and u is an effective parameter. While the first term of the above Ginzburg-Landau Hamiltonian expresses the tendency of phase separation, the second term accounts for the frustration introduced by the Coulomb interaction. This Hamiltonian is minimized in the Fourier space for a nonzero wave vector whose modulus is q_0. Thus, considering the low-energy fluctuation modes, we can expand Eq. (21) around its minimum to obtain the effective functional,

$$\mathcal{H} = \frac{1}{2} \int d^2 x d^2 x' \rho(\mathbf{x}) \mathcal{C}_0^{-1}(\mathbf{x}, \mathbf{x}') \rho(\mathbf{x}) + \frac{u}{4} \int d^2 x \rho^4(\mathbf{x})$$

$$\mathcal{C}_0^{-1}(\mathbf{x}, \mathbf{x}') = \frac{1}{(2\pi)^2} \int d^2 k \frac{e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')}}{\tau_0 + (k - q_0)^2}. \qquad (22)$$

Note that although τ_0 in Eq. (22) is shifted with respect to τ_0 in Eq. (21), we keep the same symbol to simplify the notation. This model was first studied by Brazovskii in the context of cholesteric liquid crystals[24] and has been employed to describe a variety of other physical systems with inhomogeneous states from "hard matter" ones, such as dipolar ferromagnets,[33,34] to "soft matter" systems, such as diblock copolymers[35] and microemulsions.[36] The striped phase with modulation vector $\mathbf{q_0}$, which has smectic order, is a thermodynamic stable state of the model. In two dimen-
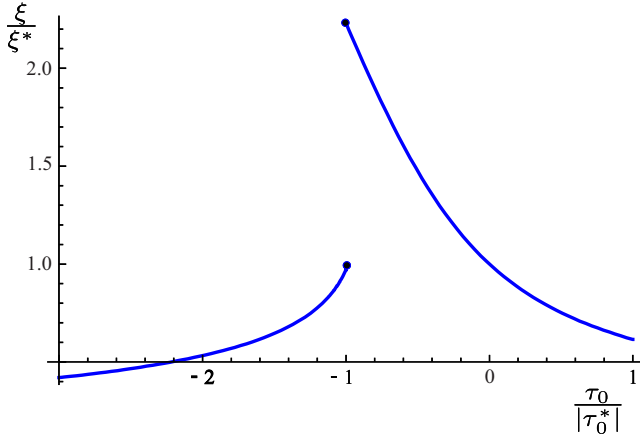
FIG. 1. (Color online) Correlation length ξ (in units of the correlation length ξ^* of the smectic phase at the reduced transition temperature τ_0^*) as function of the reduced temperature τ_0.

sions, extra external potentials are necessary in order to ensure the stability of this smectic phase against elastic fluctuations—such as crystalline fields[37] or pinning centers, for example.

Since we are interested in the main qualitative transport properties of the system, we use the self-consistent mean-field solution of Eq. (22). The details of this method are presented elsewhere.[24,34,38,39] It predicts a fluctuation-induced first-order transition from the smectic state $\langle\rho(\mathbf{x})\rangle = 2A\cos(\mathbf{q_0}\cdot\mathbf{x})$ to the liquid-disordered phase. The temperature for which their free energies become equal is $\tau_0^* \approx -(3\sqrt{2}q_0u)^{2/3}$, which we will consider the transition temperature hereafter (this is valid for an adiabatic change in the temperature). At this point, there is a jump in the correlation length of the system ξ, as sketched in Fig. 1. We consider the stripes frozen in the diffusive electron reference frame; hence, substituting the self-consistent solution of Eq. (22) in Eq. (15) yields, for the conductivity of the liquid-disordered and smectic phases,

$$\sigma_{\alpha\alpha}^{\text{liq}} \simeq \sigma_0\left\{1 - \frac{g^2}{4}(q_0\xi)\right\}$$

$$\sigma_{\alpha\alpha}^{\text{smc}} \simeq \sigma_0\left\{1 - \frac{g^2}{4}(q_0\xi)\left[1 + 4\left(\frac{\xi^*}{\xi}\right)^3\cos^2\theta_\alpha\right]\right\}, \quad (23)$$

where ξ is the correlation length, ξ^* is its value at the transition temperature for the smectic phase, and θ_α is the angle between the direction taken to measure the conductivity and the direction of the stripes modulation $\mathbf{q_0}$. As expected, the conductivity is intrinsically anisotropic and it is greater when measured parallel to the stripes ($\theta_\alpha = \frac{\pi}{2}$) than when it is measured perpendicular to them ($\theta_\alpha = 0$) $\sigma_\parallel^{\text{smc}} > \sigma_\perp^{\text{smc}}$. Moreover, this anisotropy increases as the temperature is lowered due to the decrease in ξ. Another manifestation of the thermodynamic behavior in the transport properties is the occurrence of a jump in the conductivity at the transition temperature, as shown in Fig. 2.

Not only does the module of the jump $\Delta\sigma(\theta_\alpha) = \sigma_{\alpha\alpha}^{\text{liq}}|_{\tau_0^*} - \sigma_{\alpha\alpha}^{\text{smc}}|_{\tau_0^*}$ depend on the angle between the external current
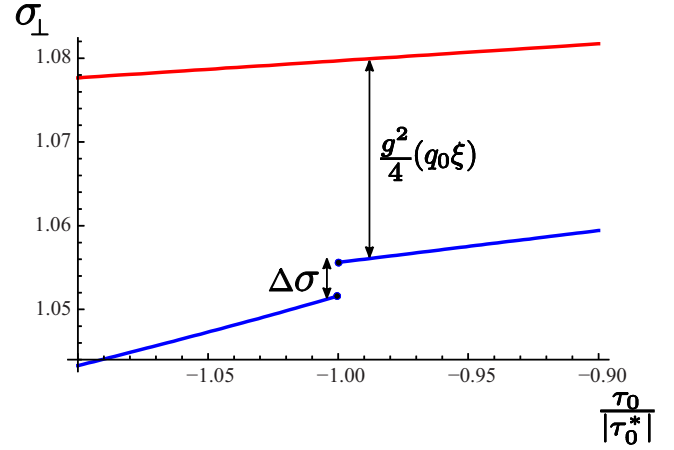
and the stripes modulation but also its sign. In Fig. 3, we plot $\Delta\sigma(\theta_\alpha)$ as a function of θ_α and see that while its sign is positive when the conductivity is measured perpendicular to the stripes, it becomes negative when the measurement is made along the stripes.

This is in agreement with the general discussion carried out in the beginning of Sec. III. The negative sign is a consequence of the fact that the conductivity measured parallel to the stripes ($\theta_\alpha = \frac{\pi}{2}$) probes the fluctuations of the order parameter, which are greater in the disordered phase, as it can be noted directly from the behavior of the correlation length presented in Fig. 1. When the conductivity is measured perpendicular to the stripes ($\theta_\alpha = 0$), even though fluctuations still contribute to the conductivity, there is an extra contribution coming from the order-parameter amplitude, which is nonzero only in the ordered phase. The positive sign is a balance between these two contributions, meaning that the latter, which is proportional to ξ^{-2}, is greater than the first, proportional to ξ.

In the low-temperature orthorhombic (LTO) phase of the layered nickelates $Nd_{1-x}Sr_xNiO_4$, a sudden upturn in the resistivity was observed at the charge-ordering temperature.[2,3]

FIG. 2. (Color online) Conductivity $\sigma_\perp$ measured perpendicular ($\theta_\alpha = 0$) to the stripes direction (blue line) as function of the reduced temperature τ_0 (in units of the transition temperature modulus $|\tau_0^*|$). An intrinsic insulating behavior was considered for σ_0 (red line).
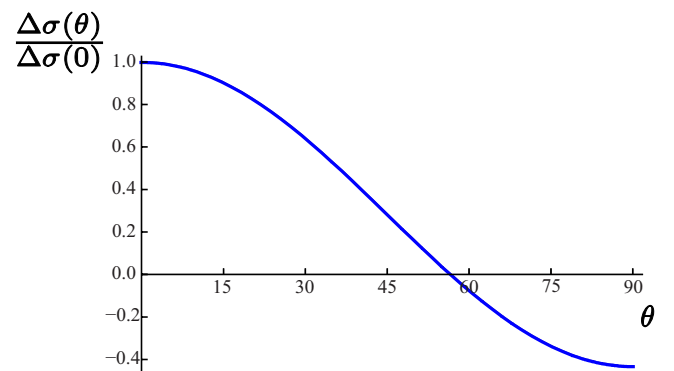

FIG. 3. (Color online) Conductivity jump $\Delta\sigma(\theta)$ as function of the angle θ between the modulation direction $\mathbf{q_0}$ and the direction taken to measure the conductivity (in degrees).

The external current in the experiment of Hücker *et al.*[3] was applied perpendicular to the direction of the stripes and this sudden change could be the result of a jump rounded by the disorder.[40] It would be interesting to realize an experiment where the conductivity is measured along the stripes direction in order to check if an opposite change in the resistivity takes place. This would provide additional criteria to check the validity range of this hydrodynamic transport model and send light on the fluctuation spectrum in a state of mesoscale ordering.

In another class of nickelates, the $La_2NiO_{4+\delta}$, experiments probing resistivity fluctuations indicated no jump but also suggested a strong coupling to sample disorder, giving rise to slow stripe dynamics.[41] The current was applied perpendicular to the stripes direction. For slow sample cooling, however, jumps were observed below the transition temperature inside the smectic phase with an increase in the resistivity. This was attributed to changes in the modulation wave-vector modulus q_0 due to the coupling to the lattice, in accordance with neutron-diffraction measurements that verified a decrease in q_0.[42] A possible explanation is given in the context of the present model: considering a reduction in the wave-vector modulus $q_0' = \eta q_0$ with $\eta < 1$ and solving the resulting self-consistent Brazovskii equations, we obtain $\sigma_\perp^{smc}(q_0') < \sigma_\perp^{smc}(q_0)$ for η close to 1 and for any temperature below the transition one. Although other mechanisms could also explain this decrease in the conductivity, such as a temperature-dependent contrast g, for example, it is worth pointing out that our model is consistent with this experimental observation.

IV. TRANSPORT IN THE NEMATIC PHASE

In Sec. III, we implicitly assumed the underlying presence of crystalline fields and/or pinning disorder in the system, in order to keep the smectic phase stable against low-energy fluctuation modes of the stripes. In the absence of such stabilizing mechanisms, it is well known that at any finite temperature, the clean two-dimensional smectic phase is unstable toward the formation of a nematic phase due to elastic fluctuations of the stripes walls.[26] Therefore, the nematic phase can be conceived as a melted smectic phase, where the stripes have no longer true positional order. Since two is the lower critical dimension of the resulting system, the 2D nematic state has only quasi-long-range orientational order, which is lost above the Kosterlitz-Thouless transition temperature T_{KT} where pairs of disclinations unbind, driving the system to the isotropic liquid phase.

Fluctuation modes of stripe walls other than thermally excited ones can also melt the smectic phase. For example, random-field disorder[18] and quantum fluctuations[1] are able to drive the system to a nematic state even at zero temperature. Alternatively, the quantum electronic nematics can be envisaged not as a result of the quantum melting of the smectic phase but as a consequence of quadrupolar Pomeranchuk instabilities of the Fermi surface of the isotropic liquid phase.[25,43] Here, we will not consider the role of quantum fluctuations nor the presence of disorder but will follow Toner and Nelson's description of the nematic phase, taking into account only thermally excited elastic fluctuations.[26] This is fully consistent with our hydrodynamic approach where the dephasing length is assumed short as the system is considered at finite T.

Without loss of generality, we consider stripes modulated along the $\hat{x}$ direction and substitute the order parameter $\rho(\mathbf{x}) = 2A \cos\{q_0[x + u(x, y)]\}$ in Eq. (22), where the displacement field $u(\mathbf{x})$ describes the elastic fluctuations. The resulting elastic Hamiltonian is given, in the harmonic approximation, by

$$\mathcal{H}_{elastic} = \frac{1}{2} B \int d^2x \left[\left(\frac{\partial u}{\partial x} \right)^2 + \lambda^2 \left(\frac{\partial^2 u}{\partial y^2} \right)^2 \right], \quad (24)$$

with $B = A^2 q_0^2$ and $\lambda^{-1} = 2q_0$. In two dimensions, explicit calculations of the mean value and of the correlation function of the order parameter yield $\langle \rho \rangle = 0$ and

$$\langle \rho(\mathbf{x})\rho(0) \rangle \propto \begin{cases} \exp\left[-\frac{q_0^2}{B} \left(\frac{|x|}{4\pi\lambda} \right)^{1/2} \right] & \text{for } \lambda x \gg y^2 \\ \left[\exp - \frac{q_0^2}{4B\lambda} |y| \right] & \text{for } \lambda x \ll y^2, \end{cases} \quad (25)$$

meaning that the system has no true positional order. After taking into account the roles of two kinds of topological defects, dislocations and disclinations, it follows that the elastic Hamiltonian describing the melted state is the Frank Hamiltonian[26]

$$\mathcal{H}_{Frank} = \frac{1}{2} \int d^2x \{ K_1 (\nabla \cdot \mathbf{N})^2 + K_3 [\mathbf{N} \times (\nabla \times \mathbf{N})]^2 \}, \quad (26)$$

which describes the elastic properties of a 2D nematic liquid crystal. In the expression (26), the unit vector $\mathbf{N}$ denotes the nematic director and the Frank constants K_1 and K_3 are functions of the elastic parameters B and λ, as well as of the energy necessary to excite an isolated dislocation E_D.

The role of the topological defects is fundamental to the understanding of the thermodynamics of the resulting nematic phase. As explained above, the unbinding of disclination pairs induces a Kosterlitz-Thouless transition to the isotropic liquid phase. Moreover, since isolated dislocations have a finite excitation energy E_D, a new length scale ξ_D is introduced in the system below T_{KT}. Such a length scale denotes the correlation length of isolated dislocations that proliferate in the system as the temperature is increased and is given by $\xi_D = n_D^{-1/2}$, where $n_D = d^{-2} \exp(-\frac{E_D}{k_B T})$ is the dislocations density and $d \propto q_0^{-1}$ is the stripes mean width. For lengths greater than ξ_D the system has the same properties of a nematic liquid crystal, while for smaller lengths there is smectic order decorrelated by elastic fluctuations only. Therefore, one can consider the nematic phase as a set of finite-size smectic blobs,[26] where each blob has orientational order given by the average of the orientation of the stripes enclosed, as sketched in Fig. 4.

We can now apply the formulae of Sec. III to calculate the conductivity of an individual blob $\sigma_{\alpha\alpha}^{blob}$. Inside it, although
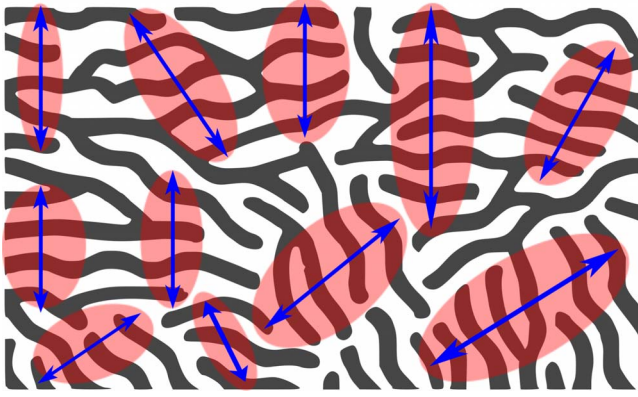
FIG. 4. (Color online) Schematic representation of the nematic phase originated from the melting of the smectic state due to thermally excited elastic fluctuations. The blobs comprehend regions free of topological defects enclosing stripes that are decorrelated by elastic fluctuations only. The director of each blob is indicated by a double arrow.

density fluctuations remain small as long as $\tau_0 < \tau_0^*$, elastic fluctuations are relevant. While a diffusive electron takes the typical time $t_{el} \sim \sigma_0^{-1} \xi_D^2$ to cross the blob, phonon fluctuations propagate with a typical time scale $t_{ph} \sim v^{-1} \xi_D$, where v is a sound velocity. Therefore, we have to substitute the order-parameter mean value $\langle \rho(\mathbf{k}) \rangle = A[\delta(\mathbf{k}-\mathbf{q_0}) + \delta(\mathbf{k}+\mathbf{q_0})]e^{-W}$, where $W = e^{E_D/3k_B T}/(2^{2/3}\pi^{1/3}A^2)$ is the Debye-Waller factor of the finite-size blob, in the annealed limit of the macroscopic conductivity [Eq. (16)]. We obtain

$$\sigma_{\alpha\alpha}^{\text{blob}} \simeq \sigma_0(1 - 2g^2 A^2 e^{-2W} \cos^2 \theta_\alpha), \qquad (27)$$

where θ_α is the angle between the direction taken to measure the conductivity and the blob director (we are following the convention that the director is perpendicular to the stripes orientation inside each blob). This procedure is valid for intermediate temperatures only, where W is small and the elastic fluctuations $|\delta\rho_{el}| = \sqrt{e^{2W}-1}\langle\rho_{el}\rangle$ are not so large.

To obtain the system's macroscopic conductivity, the fluctuations of the nematic director inside each blob have to be taken into account. We can consider them slow in the electron reference frame. Thus, while taking the annealed limit to calculate the conductivity inside each blob, we have to take the quenched limit to calculate the conductivity of the whole system, averaging over all blobs. We obtain

$$\sigma_{xx}^{\text{nem}} \simeq \bar{\sigma}_0(1 - \bar{g}\langle\cos 2\zeta\rangle)$$

$$\sigma_{yy}^{\text{nem}} \simeq \bar{\sigma}_0(1 + \bar{g}\langle\cos 2\zeta\rangle) \qquad (28)$$

for the two opposite situations where the direction taken to measure the conductivity is parallel ($\langle\theta_\alpha\rangle = 0$: $\hat{x}$ axis) or perpendicular ($\langle\theta_\alpha\rangle = \pi/2$: $\hat{y}$ axis) to the mean orientation of the nematic director. In the expression (28), $\zeta = \theta_\alpha - \langle\theta_\alpha\rangle$ denotes the director deviation from the mean value. Furthermore, it holds that $\bar{\sigma}_0 = \sigma_0(1 - g^2 A^2 e^{-2W})$ and $\bar{g} = g^2 A^2 e^{-2W}$ (to the second order in the original g).

It is remarkable that the nematic order parameter $\langle\cos 2\zeta\rangle$ appears spontaneously in the conductivity expression, even though it was not assumed any coupling between the micro-

scopic conductivity and the nematic order parameter. Such a relation can be cast in a more formal way by considering the tensorial nematic order parameter in two dimensions $\langle S_{\alpha\beta}\rangle = S(N_\alpha N_\beta - \frac{1}{2}\delta_{\alpha\beta})$ with $S = \langle 2\cos^2\zeta - 1\rangle$. It is clear from Eq. (28) that the anisotropic conductivity can be written as $\sigma_{\alpha\beta}^{\text{nem}} \simeq \bar{\sigma}_0(1 - \bar{g}\langle S_{\alpha\beta}\rangle)$.

Another direct consequence of Eq. (28) is that the conductivity anisotropy is proportional to the nematic order parameter $\sigma_{yy}^{\text{nem}} - \sigma_{xx}^{\text{nem}} = 2\sigma_0 g^2 A^2 e^{-2W}\langle\cos 2\zeta\rangle$ in agreement with symmetry arguments.[27] By comparing this expression with the one referring to the conductivity anisotropy of the smectic phase [Eq. (23)], we note that in the present case the anisotropy is much smaller due to the Debye-Waller factor e^{-2W}. Moreover, this result agrees exactly with the relation

$$\left(\frac{r+1}{r-1}\right)\left(\frac{R_\perp - R_\parallel}{R_\perp + R_\parallel}\right) = \langle\cos 2\zeta\rangle \qquad (29)$$

found for $T=0$ by Carlson *et al.*,[18] with the same prefactors [in our case, $r \equiv \dfrac{R_\perp(\langle\cos 2\zeta\rangle \to 1)}{R_\parallel(\langle\cos 2\zeta\rangle \to 1)} = \dfrac{1+\bar{g}}{1-\bar{g}}$]. In that work, the disordered electronic nematics was mapped on the random-field Ising model (RFIM) and the conductivity was obtained numerically through a random resistor network approach. Although the two systems are very different, since ours is clean whereas the one investigated by Carlson *et al.*[18] is disordered, this outcome seems to be a consequence of the existence of clusters with short-range smectic order inside the nematic phase—the thermally excited blobs in our model and the disorder-generated nematic patches in the other work.

Such a linear relation between a macroscopic quantity and the thermodynamic order parameter can be very useful in the investigation of the orientational ordering of nematic phases in the cuprates, especially through transport measurements. In fact, a recent experiment[11] on YBa$_2$Cu$_3$O$_{6.45}$ suggested a linear relation between the compound conductivity anisotropy and the spectral weight of the low-energy anisotropic spin fluctuations. Still regarding possible experimental implications, we also point that due to the large elastic fluctuations intrinsic to the system's dimensionality, deviations from the averaged conductivity are expected for small systems in what would be manifested as noise in time-series measurements.

Before finishing this section, an important remark regarding the mean value $\langle\cos 2\zeta\rangle$ should be made. Strictly speaking, in two dimensions this average is zero in the thermodynamic limit due to Mermin-Wagner theorem (see, for instance, Chaikin and Lubensky).[28] However, in real layered systems, finite-size effects as well as small interlayer couplings are able to stabilize the nematic phase, granting a nonvanishing value for the order parameter.

V. CONCLUDING REMARKS

We presented a hydrodynamic transport theory that can provide important tools in the investigation of electronic phases with smectic and nematic symmetries, rendering explicit relations between macroscopic transport quantities and the microscopic order parameter. Particularly, we considered

doped-layered transition-metal oxides, using the Brazovskii model to describe the thermodynamics of the low-energy charge modes of their smectic phases. The directional dependence of the sign of the conductivity jump was shown to be a manifestation of the Brazovskii fluctuation spectrum and a general characteristic of the hydrodynamic transport model, constituting an interesting criterion to decide on the applicability of the model to describe the observed static charge-striped phases.

The finite temperature electronic nematics was conceived as a smectic Brazovskii phase melted by thermally excited elastic fluctuations of the stripes walls, following the approach of Toner and Nelson.[26] Not only does the nematic order parameter appear explicitly and spontaneously in our formalism, but it is also shown to be linearly proportional to the conductivity anisotropy, following the same relation found numerically by Carlson et al.[18] for the case of the disordered electronic nematics at zero temperature.

In the context of nematic phases, it would be interesting to investigate other excitations that are also able to melt the smectic phase in a nematic state, such as quantum and disorder-induced fluctuations.[1,18] Moreover, additional studies on out-of-equilibrium properties of the electronic nematics, particularly on the connection between fluctuations of the smectic clusters and transport time-series measurements, would also be desirable to provide a richer picture of the problem. Finally, applications of the general formalism to systems other than doped transition-metal oxides can also be envisaged, for example, for the low-temperature-striped phases that appear in quantum Hall systems[25,27,43] and for the inhomogeneous phases found in spin glasses and Mott insulators.[19]

*rafaelmf@ameslab.gov

[1] S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature (London) **393**, 550 (1998).

[2] S. W. Cheong, H. Y. Hwang, C. H. Chen, B. Batlogg, L. W. Rupp, Jr., and S. A. Carter, Phys. Rev. B **49**, 7088 (1994).

[3] M. Hücker, M. v. Zimmermann, and G. D. Gu, Phys. Rev. B **75**, 041103(R) (2007).

[4] S. Mori, C. H. Chen, and S. W. Cheong, Nature (London) **392**, 473 (1998).

[5] J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, Nature (London) **429**, 534 (2004).

[6] M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **82**, 394 (1999).

[7] R. A. Borzi, S. A. Grigera, J. Farrell, R. S. Perry, S. J. S. Lister, S. L. Lee, D. A. Tennant, Y. Maeno, and A. P. Mackenzie, Science **315**, 214 (2007).

[8] Y. Ando, K. Segawa, S. Komiya, and A. N. Lavrov, Phys. Rev. Lett. **88**, 137005 (2002).

[9] A. N. Lavrov, I. Tsukada, and Y. Ando, Phys. Rev. B **68**, 094506 (2003).

[10] J. A. Bonetti, D. S. Caplan, D. J. Van Harlingen, and M. B. Weissman, Phys. Rev. Lett. **93**, 087002 (2004).

[11] V. Hinkov, D. Haug, B. Fauque, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer, Science **319**, 597 (2008).

[12] P. G. de Gennes, *The Physics of Liquid Crystals* (Clarendon, Oxford, 1974).

[13] S. Kirkpatrick, Rev. Mod. Phys. **45**, 574 (1973).

[14] G. Hurvits, R. Rosenbaum, and D. S. McLachlan, J. Appl. Phys. **73**, 7441 (1993).

[15] M. Mayr, A. Moreo, J. A. Vergés, J. Arispe, A. Feiguin, and E. Dagotto, Phys. Rev. Lett. **86**, 135 (2001).

[16] M. M. Parish and P. B. Littlewood, Nature (London) **426**, 162 (2003).

[17] P. J. M. Bastiaansen and H. J. F. Knops, J. Phys. A **30**, 1791 (1997).

[18] E. W. Carlson, K. A. Dahmen, E. Fradkin, and S. A. Kivelson, Phys. Rev. Lett. **96**, 097003 (2006).

[19] S. Papanikolaou, R. M. Fernandes, E. Fradkin, P. W. Phillips, J. Schmalian, and R. Sknepnek, Phys. Rev. Lett. **100**, 026408 (2008).

[20] J. A. Blackman, J. Phys. C **9**, 2049 (1976).

[21] A. Weinrib, Phys. Rev. B **29**, 387 (1984).

[22] V. J. Emery and S. A. Kivelson, Physica C **209**, 597 (1993).

[23] U. Löw, V. J. Emery, K. Fabricius, and S. A. Kivelson, Phys. Rev. Lett. **72**, 1918 (1994).

[24] S. A. Brazovskii, Zh. Eksp. Teor. Fiz. **68**, 175 (1975).

[25] V. Oganesyan, S. A. Kivelson, and E. Fradkin, Phys. Rev. B **64**, 195109 (2001).

[26] J. Toner and D. R. Nelson, Phys. Rev. B **23**, 316 (1981).

[27] E. Fradkin, S. A. Kivelson, E. Manousakis, and K. Nho, Phys. Rev. Lett. **84**, 1982 (2000).

[28] P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, Cambridge, 1995).

[29] M. J. Stephen, Phys. Rev. B **17**, 4444 (1978).

[30] J. Schmalian and P. G. Wolynes, Phys. Rev. Lett. **85**, 836 (2000).

[31] H. Westfahl, J. Schmalian, and P. G. Wolynes, Phys. Rev. B **64**, 174203 (2001).

[32] V. J. Emery, S. A. Kivelson, and J. M. Tranquada, Proc. Natl. Acad. Sci. U.S.A. **96**, 8814 (1999).

[33] T. Garel and S. Doniach, Phys. Rev. B **26**, 325 (1982).

[34] R. M. Fernandes and H. Westfahl, Jr., Phys. Rev. B **74**, 144421 (2006).

[35] G. H. Fredrickson and E. Helfand, J. Chem. Phys. **87**, 697 (1987).

[36] S. Wu, H. Westfahl, Jr., J. Schmalian, and P. G. Wolynes, Chem. Phys. Lett. **359**, 1 (2002).

[37] A. Abanov, V. Kalatsky, V. L. Pokrovsky, and W. M. Saslow, Phys. Rev. B **51**, 1023 (1995).
[38] P. C. Hohenberg and J. B. Swift, Phys. Rev. E **52**, 1828 (1995).
[39] H. Westfahl and J. Schmalian, Phys. Rev. E **72**, 011806 (2005).
[40] Y. Imry and M. Wortis, Phys. Rev. B **19**, 3580 (1979).
[41] A. Pautrat, F. Giovannelli, and N. Poirot, Phys. Rev. B **75**, 125106 (2007).
[42] P. Wochner, J. M. Tranquada, D. J. Buttrey, and V. Sachan, Phys. Rev. B **57**, 1066 (1998).
[43] K. Sun, B. M. Fregoso, M. J. Lawler, and E. Fradkin, Phys. Rev. B **78**, 085124 (2008).